

An Image denoising method combining dt-cwt and Fourth-order PDE

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Abstract: It is proved that image gray level tends to be piecewise constant in the low order partial differential equation denoising model. Considering the applications of wavelet multi-scale decomposition in image processing, a kind of remote sensing image denoising model combining dual tree complex wavelet transform and fourth order partial differential equation are put forward. Firstly, the noise images are multiscalely decomposed by DT-CWT. Then the low frequency components are reserved and noise in high frequency components of other layer are removed by the fourth order PDE model. Finally, the high and low frequency components of the corresponding layer are reconstructed to get the final image. According to the denoising experiments results of the ZY-3 satellite images which have different noise intensities proved that the average PSNR of results by proposed model increases 1~2 DB and the structural similarity increases as. The denoising model proposed can effectively preserve the image details while removing noise.

Keywords: dual tree complex wavelet transform ; Partial differential equation remote sensing image; denoising; diffusion model

due to remote sensing imagery and image transfer, save save, decoding inevitably a lot of dry scrambling, studying image denoising method to improve image readability and discrimination disjunction, in remote sensing image segmentation, object category, Change Detection for applications with significant m.

The contradiction between image smoothing and edge protection enhancement

sex is always a key and difficult point in the field of image denoising. based on partial differential equation PDE (Partial differential equation) Go method of noise because of its good local adaptability, The model establishes flexibility and other features become important imaging tools H. document 4] Submit Classic PM (perona-Malik) Nonlinear diffusion model, the process of smoothing the image to a "" PDE

Solution, and use the grayscale gradient as a that affects the smooth diffusion of the image Important factor. document 5] on PM further of the model Improve, make diffusion go only along the direction perpendicular to the gradient vector of the image on, de-noising with strong edge protection, but low orders An iterative calculation of PDE causes the image to tend to segment constants and produce Health " Ladder phenomenon "6. document 7] using Laplace operator mode value instead of gradient modulus construct energy function, proposed One Four-step PDE image denoising method, can effectively suppress Ladder Effect should. literature 8] general in image processing PDE makes the category and plans the PDE numerical

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solution process. When the image represents A as a continuous signal, PDE can be considered in a small

neighborhood local Filtered iterations , This property makes the PDE have a good compatibility features , that are combined with other image processing methods swing Their advantages H. document [10-11] Research PDE with wavelet transform combined image denoising method and proposed improved denoising model . document [a]Indicates that most of the image noise is divided into fabric after wavelet decomposition High frequency sub band , But because traditional two-dimensional from scatter wavelet transform DWT (discrete Wavelet Transform) Save problems with translation sensitivity and incomplete refactoring , In the Case of a high-resolution remote sensing image with a complex noise ,, does not describe the well image mutation and details . document [] Proposed double-tree complex wavelet transform dt~cwt (dual~treeComplex Wavelet Transform) not only keep DWT Good time-frequency analysis can Force , with an approximate translation invariance , Good orientation Optional , Limited data redundancy and benefits of complete refactoring .

This article presents the _ a four-order with a dual-tree complex wavelet transform partial differential equation image denoising Model . first with DT~CWT Multi-scale decomposition of noise images , To decompose images into different The detail components and feature components in the scale window , on decomposed High frequency components using the four-order PDE denoising model for denoising ; and then refactoring the corresponding layer high , low-frequency Components , has reached the remove image noise protects image details and edge effects .

1. basic theory

1.1 Double tree complex wavelet transform

- Dimension DT ~ CWT exploded schematic diagram 1 (a) shows , its in $h_0(n)$, $h_1(n)$ the represents a common orthogonal filter to ; $G_0(n)$, $G_1(n)$ to indicate conjugate integral filters to ; " | 2 The represents the interval mining like . The Basic implementation of the is to make the complex wavelet real and imaginary parts of the build, separate from , through two sets of parallel real number filter groups $U_0(n)$, $M(n)$ } and $\{g_0(n)$, $g_1(n)\}$,- Tree (Tree a) generate duplicate

The real part of the wavelet transform coefficient , another tree (Tree b) produce virtual part change factor . per-layer decomposition to get The high frequency components of the 6 Direction $\{\pm 15^\circ, \pm 45^\circ, \pm 75^\circ\}$ and a low-frequency component . One-dimensional DT- CWT refactoring diagram 1(b) is shown in . Same as two trees , I. tree Reverse transform real part of complex wavelet , Another tree pair The Virtual part of the complex wavelet has a reverse transformation . gets two sets of solid signals , andThe takes the average of the two sets of signals as the final output .

The construction method for two-dimensional dual-tree complex wavelet transforms is similar to the - Dimension Double Tree complex wavelet construction , Two-dimensional dual-tree complex wavelets can be constructed by a wavelet tensor product by a-D two-tree complex wavelet , Concrete Constructor Law See documents] . to overcome the odd / I filter for strict The disadvantages of the linear phase cause differences in sample structures , This article will take the Kingsbury q-shift based on orthogonal transformations (quarter Sample Shift) Filter group M Two-dimensional for noisy images solution , where The decomposed low-frequency bands are lower for grayscale gradients View information , The high frequency band after decomposition contains a large number of image noise

Sound and information such as edge of image .

Four-step PDE

The basic idea of image denoising based on partial differential equations is to map the The Smoothing process in the time scale analogy to the heat in physics theconduction diffusion process . diffusion media to homogeneous in physics quality , the diffusion rate in each direction is constant constant . corresponds to smooth image for all same-sex proliferation , is the same that removes noise also loses important edge information for the image .

Perona, and so on 1990 The year presents a special that can remain marginal Anisotropic diffusion equation for the sign [4,],

$$(\frac{\partial u}{\partial t} = \text{Div} g(|\nabla u|) |\nabla u|); (x, y, 0) = u_0(x, y).$$

type : $u(x, y, t)$ for time scale t image area ; ∇u to image gradient ; $G(|\nabla u|)$ non-negative recursion for gradient modulus minus functions , the purpose is to control the speed at which images are spread . Two classic function expression to [4

$$RG(|\nabla u|) = 1 + (|\nabla u|/K)^2; (2)$$

$$JG(|\nabla u|) = \exp[-(|\nabla u|/K)^2].$$

type , K is a gradient threshold parameter , K The greater the value the smoother the intensity is Greater . in flat image area , due to $|\nabla u|$ smaller , spread plus Strong for denoising ; in Edge area $|\nabla u|$ large , diffusion The speed is smaller to suppress smooth effects . The diffusion model base in The energy functional :

$$E(u) = \int_{\Omega} f(|\nabla u|) dx. (3)$$

Image region in ; ∇u is the image gradient ; $f(\cdot) \geq 0$, monotonically increasing function and meeting $g(x) = f'(x) / x$.

PM The equation controls the diffusion speed only according to the of the image gradient modulus , on larger noise background , The image gradient has a very big uncertainty , hard to reflect real image information for images . Alvarez This is based on a "" PM Improved model 0 , its equation is

$$\{\frac{\partial u}{\partial t}(x, y, t) = \text{Div} (|\nabla u| \text{Div} (|\nabla u|^{-1} \nabla u))\}; (4)$$

$$u(x, y, 0) = u_0(x, y).$$

The diffusion model is perpendicular to the image gradient ∇u direction of for , with strong edge protection and noise removal . But because the diffusion rate of the second order PDE in homogeneous region is clear Significantly higher than edge area , image tends to score after multiple iterations Segment constant constant , To make an image spread easily by Ladder phenomenon .

the formula (3) The gradient operator in uses the Laplace algorithm for the descendant to, describe image smoothness , to get a four order partial differential denoising mode type . The constructs the following energy functionals :

$$E(u) = \int_{\Omega} f(|\nabla^2 u|) dx. (5)$$

type : ∇^2 is a Laplace operator ; $f(\cdot) > 0$ is about $|\nabla^2 u|$ monotonically increasing functions . minimize This energy function is equivalent to finding a minimal $|\nabla^2 u|$, up to to image smoothing to remove noises . So according to the Euler equation principle , image denoising problem translates to the function (5)find minimum problem .

(5) for Euler ~ Lagrange equation is

$$\nabla^2 u - |\nabla^2 u|^{-1} \text{Div} (|\nabla^2 u| \nabla u) = 0. (6)$$

consists of the $G(x) = f'(x) / x$ (6) converted to

$$\nabla^2 [G(|\nabla^2 u|) \nabla^2 u] - \nabla^2 u = 0 (7)$$

by gradient descent M , introduces a time variable T , will contain noise Image Smoothing process as an energy function for an image the process of gradual reduction over time . by type (7) available

$$\frac{\partial u}{\partial t}(x, y, t) = \text{Div} g(|\nabla^2 u|) \nabla^2 u. (8)$$

(8) is four-order PDE denoising Model . discretization solution the equation can get the result of noisy image after smooth denoising . The model can effectively suppress lower-order skewness PDE noise-generated Stair effect , while keeping the denoising effect , first degree guaranteed holds the texture feature of the image .

2. Remote Sensing image denoising model

The wavelet transform allows the detail component of the image and the feature component to be in the Time-frequency domain range to different scale Windows , to wavelet decomposition Nonlinear diffusion of high frequency bands containing large amounts of noise will be more targeted removal of image

noise ; simultaneous four-step PDE relative to Lower order PDE De-noising has a significant effect of eliminating the ladder and better to protect image details . based on the previous analysis , This article references a four-order partial differential equation with two-tree complex wavelet transform shadow image denoising model . The algorithm flow for the model is shown in the diagram 2 ..

2.1 Numerical solution to the four-order PDE

is a discretization iteration solver (8) , using the finite difference method to disperse it . Here's the Central point $u(i, j)$ Grayscale Rapp The calculation of the Russ operator takes a traditional neighborhood-based domain structure of the Symmetric difference method , neighborhood domain structure as illustrated 3 .

Select a diffusion function $g(L, v)$, The of the diffusion function Second-order difference format is

$$\Delta^2 u = \frac{1}{4} (u_{i+1, j} + u_{i-1, j} + u_{i, j+1} + u_{i, j-1} - 4u_{i, j}) / \Delta t \quad (one)$$

(8) the iteration formula after discretization is

$$u_{i, j}^{k+1} = u_{i, j}^k - \Delta t \Delta^2 u_{i, j}^k \quad \Delta t \text{ type at for discretization time step .}$$

2.2 algorithm Steps

diagram 2 , Is based on the DT ~ CWT Four-order of PDE denoising algorithm Basic steps are as follows .

1) Initialize image : set I_0 for original noise image , diagram 4 (a) and Diagram 5 (a) ; Δt indicates time after discretization step , initial value 0.02 Δt for iterations .

2) take Q-Shift Filter group for noisy remote sensing images Line two-dimensional DT ~CWT decompose , gets the 6 The High-frequency component of the direction $\theta \in \{15^\circ, 45^\circ, 75^\circ, 105^\circ, 135^\circ, 165^\circ\}$ and a low-frequency component B .

3) apply to the real and imaginary parts of a complex high-frequency band

(9) calculates the second-order difference of image grayscale .

4) Select $g(L, v) = 1 + (L, v/k)^2$ for proliferation function and select gradient thresholds based on image reality and noise level value K .

5) based on (one) Compute diffusion function Second difference .

6) apply (a) iteration to iteration count , Gets the high frequency wavelet coefficients after denoising YH .

7) High frequency components after denoising YH and wavelet low-frequency components B Double-tree complex wavelet reconstruction , get de-noising image , such as diagram 4(e) and figure 5 (e) is shown in .

3. Experiment and Analysis

to verify the validity of this method , Select a size of The like X "A ZY-3 satellite images A ,,B as experimental images , These two images contain rivers , Road , Building ,vegetation etc Rich feature information . Take advantage of Matlab tool to add different intensity Gauss noise images with improved PM model S , DWT - Four-step PDE de- noising and the combined dual-tree complex wavelet transform of this article four Order-Partial differential equation denoising model for experiment and contrast analysis ,noise The sound variance is 0.005 , 0.01, 0.02, 0.03, 0.04, 0.05. with 3 filter Noise variance to 0.03 Remote sensing image de -noising Experimental results For example, Diagram 4 and Diagram 5 shows .

from Diagram 4 and Diagram 5 to see , with improved PM Model De -noising image , Although more thoroughly removed Gaussian noise ,, But image details texture features are also severely damaged , edge anti bad Severe and distorted ; Take a pair DWT decompose high-frequency information into Line four-order PDE de -noising image contains a large number of visible noises and show _ degree of Blur ; with binding DT-CWT Four order PDE The denoising image obtained by the denoising model has a good clear degree , Building , Farmland , The details of routes and river lines are more detailed than is rich and delicate .

to objectively evaluate the results of the experiment , Select Peak Snr PSNR and structure similarity $ssim$ [1] Two indicators as image Quality Evaluation Indicator . PSNR refers to the original signal with the noise Ratio of energy between , Snr higher , The better the denoising effect ; Ssim refers to two image

similarity, The higher the value, two images degree higher, indicates that the image restores the original more completely after denoising image. To Image A and images B with the above 3 species gonnoise model after denoising, Its image quality evaluation results are as follows. 6 and diagram 7 are shown.

diagram 7 Image B denoising Results PSNR and Ssim Graph

from Diagram 6 and Diagram 7 to see, at different noise intensities Environment, binding DT ~CWT four orders PDE denoising method Get de-noising image, PSNRAverage increase 1~2 dB, and ssim refers to subscript also above improvement PMmodel denoising and DWT four orders PDE go noise. The explains that the noise removal algorithm for Gaussian noise of the client "" is better than the other two methods.

to better validate the validity of this method, To Select a real noise ZY-3 Remote sensing image of an island, with the above 3 A method of denoising image results as shown in the figure 8, "".

from Diagram 8 (a) Sea Area visible The original image is clearly fine-grained and uniform noise. take an improved PM model denoising image produces a significant step effect, Island details badly lost; Mining with DWT- four-step PDE The Sea area still has the naked eye Visible Noise clutter; with this article DT - C WT Four-order of PDE image de-noising method obtained denoising image black area more is pure, Island details Clear.

4. Conclusion

Surround image denoising, The presents the _ To combine a double-tree complex Image denoising for Four-order PDE of wavelet transform. the following two measures are used to optimize the denoising effect by The This square.

1) Multi-scale decomposition of noisy images, add fine for image section components and feature components are separated to different scales in time-frequency domain windows, with an approximate translation invariance, more Parties to selectivity and complete refactoring DT- CWT transform instead of the traditional DWT, to maximize the integrity of the image sex is not corrupted.

2) The High frequency band after the image is decomposed to four steps PDE de-noising, Converts the denoising process of an image to a higher-order PDE for Solver, overcome traditional low order PDE the produces a clear when denoising Explicit stair effects and other defects, to better protect diagram

Details of the image.

The disadvantage of the is that the noise model of this paper is on salt and pepper noises, Spot point noise noise denoising with _ Limitations. under - Step research should be directed at the characteristics of different noises to the partial differential equation The operators and diffusion functions of the are improved accordingly. at the same time, biased the setting of parameters in the process of partial-equation denoising and the selection of iteration times options have a significant effect on imagedenoising. How to adapt to select the appropriate parameters and iterations in the partial differential equation element requires further study.

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