Statistics and Analysis of Similar Figures Questions in Zhejiang Senior High School Entrance Examination in Recent Five Years——From the Perspective of Core Literacy

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Abstract: Based on the perspective of core literacy, this paper makes a statistical analysis of the similar figures test questions in the eight high school entrance examinations in Zhejiang Province in recent five years, and analyzes the comprehensive difficulty of each year’s test questions from five dimensions of cognitive level, background, operation, reasoning and knowledge content with the help of comprehensive difficulty model. The results show that: the core literacy of the similarity test mainly includes logical reasoning, intuitive imagination and mathematical operation; Background comprehensive difficulty is lower than the other four dimensions, and the difficulty of the five dimensions is unbalanced, but the overall difficulty of the similar figure test is larger. Based on this finding, this paper analyzes the reasons for the difficulty of the test from the perspective of core literacy, and finally puts forward teaching strategies to help the development and improvement of students’ core literacy.

Keywords: Similar Figures; Core Literacy; Zhejiang High School Entrance Examination; Statistical Analysis

Introduction
The knowledge of similar triangles is the content of the first volume of junior high school mathematics textbook of Zhejiang Education Press. The requirements of “mathematics curriculum standard for compulsory education” for similar triangles involve the level of understanding, judgment, application, etc. At the same time, similar triangles are also “frequent visitors” in the final questions of senior high school entrance examination in Zhejiang Province. Therefore, based on the statistical analysis of the senior high school entrance examination questions of eight similar triangles in Zhejiang Province in recent five years, this paper studies the examination content and core literacy embodiment of similar triangles from a qualitative point of view, and analyzes the difficulty of similar triangles from a quantitative point of view. Through the qualitative and quantitative research of the test questions, it provides suggestions and reference for teachers’ teaching and students’ learning.

1. Examination content analysis

Based on the statistics of similar figure test questions in eight places of Zhejiang Province from 2017 to 2021, some results are shown in Table 1 below. The judgment of test content is based on the answers and analysis of senior high school entrance examination questions published by the government.

<table>
<thead>
<tr>
<th>Region</th>
<th>Number of questions</th>
<th>Total score (points)</th>
<th>Comprehensive difficulty</th>
<th>Core literacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hangzhou</td>
<td>7 (A) 10 (B)</td>
<td>10/15.75</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
<tr>
<td>Daxi</td>
<td>7 (A) 10 (B)</td>
<td>12/10.29</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
<tr>
<td>Nonjiren</td>
<td>6 (A) 9 (B)</td>
<td>10/12.16</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
<tr>
<td>Jiaxing</td>
<td>5 (A) 8 (B)</td>
<td>9/11.68</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
<tr>
<td>Jiaxing</td>
<td>4 (A) 6 (B)</td>
<td>8/9.64</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
<tr>
<td>Jiaxing</td>
<td>3 (A) 5 (B)</td>
<td>6/7.54</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
<tr>
<td>Huaxi</td>
<td>2 (A) 4 (B)</td>
<td>5/6.91</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
<tr>
<td>Huaxi</td>
<td>1 (A) 3 (B)</td>
<td>4/5.88</td>
<td>Determination and properties of similar triangles</td>
<td>7 (A) 10 (B)</td>
</tr>
</tbody>
</table>

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Note: A represents choice question; B represents fill in the blanks; C represents answer question; core literacy a represents logical reasoning; b represents intuitive imagination; c represents mathematical operation.

It can be seen from the statistical table that the similar figure questions may appear in all types of questions, and the similar figure will be tested in the answer questions, and the scores of similar figure are higher in the high school entrance examination. This result proves the necessity of this study again.

In addition to the examination of question types and scores, the content of similar figure examination is basically to find and construct similar triangles, and the judgment and nature of similar triangles. The search for similar triangles must test the core literacy of intuitive imagination, the judgment of similar triangles must test the core literacy of logical reasoning, and the test of mathematical operation in the core literacy is reflected in the application of its nature after the judgment of similar triangles.

Based on the above analysis, the systematicness of the content and literacy of similar subjects can also explain the high frequency and high score of the high school entrance examination.

2 Analysis on the difficulty of similar figure test
2.1 Introduction of comprehensive difficulty model
The comprehensive difficulty model of Bao Jiansheng\(^1\) is used to analyze the comprehensive difficulty of the test questions in this study. The model divides the difficulty of mathematics test questions into five dimensions: Inquiry (hereinafter referred to as cognitive level), background, operation, reasoning and knowledge content. See Table 2 below. The cognitive level is divided into three levels: memorizing, understanding and exploring. The score of each level is 1, 2 and 3 in turn; Background is divided into four levels: no knowledge background, personal life, public knowledge and scientific situation. The score of each level is 1, 2, 3 and 4 in turn; The operation is divided into four levels: no operation, numerical operation, simple symbolic operation and complex symbolic operation. The score of each level is 1, 2, 3 and 4 in turn; The reasoning dimension includes no reasoning, one-time reasoning, two-time reasoning and so on. The score of each level is 1, 2, 3 and so on, which increases by 1 in turn. If 3 times of reasoning, the score of reasoning dimension is 4. After \(n(n\) is non negative integer) times of reasoning, the score of reasoning dimension is \(n+1\); Similarly, if the question contains \(m\) knowledge points, the score of the dimension is \(m\).

<table>
<thead>
<tr>
<th>Difficulty factor</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cognitive level</td>
<td>Memorize</td>
<td>Understand</td>
<td>Explore</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Background</td>
<td>No real background</td>
<td>Personal life</td>
<td>Common sense</td>
<td>Scientific context</td>
<td></td>
</tr>
<tr>
<td>Operation</td>
<td>No operation</td>
<td>Numerical operation</td>
<td>Simple symbolic operation</td>
<td>Complex symbolic operations</td>
<td></td>
</tr>
<tr>
<td>Reasoning</td>
<td>No reasoning</td>
<td>One step reasoning</td>
<td>Two step reasoning</td>
<td>Three step reasoning</td>
<td></td>
</tr>
<tr>
<td>Knowledge content</td>
<td>A knowledge point</td>
<td>Two knowledge points</td>
<td>Three knowledge points</td>
<td>Four knowledge points</td>
<td></td>
</tr>
</tbody>
</table>

This study carries out the comprehensive difficulty statistics and calculation of each dimension according to the year, that is, from 2017 to 2021, the senior high school entrance examination questions in eight places of Zhejiang Province are graded according to the five dimensions of cognitive level, background, operation, reasoning and knowledge content. The scoring standard is shown in Table 2 above. At the same time, the scores are given in combination with the official reference answers and analysis to ensure the accuracy of the statistical results.

The difficulty calculation formula \(d_l\) is as follows:

\[
d_l = \frac{\sum n_{ij}a_{ij}}{\sum j n_{ij}} (i = 1,2,3,4,5; j = 1,2,3)\]

Among them, \(i\) represents the \(i\) th dimension, \(j\) represents the \(j\) th level, \(n_{ij}\) represents the number of the \(j\) th level of the \(i\) th dimension similar test questions in a certain year, and \(a_{ij}\) represents the grade score of the \(j\) th level of the \(i\) th dimension. \(\Sigma n_{ij}\) represents the total number of similar items in eight places of Zhejiang Province in a certain year.

2.2 Comprehensive difficulty results and analysis
The results of statistical analysis based on the comprehensive difficulty model are shown in Figure 4 below. For the convenience of comparison, this paper transforms the difficulty value into the relative difficulty value, which is the ratio of the difficulty to the highest difficulty of the dimension. As can be seen from the figure below, the difficulty of similar subjects in the 2017-2021 senior high school entrance examination in Zhejiang Province is relatively stable. However, the relative difficulty of the background is significantly lower than the other four dimensions, and the similar questions are basically pure mathematical questions without other backgrounds. The cognitive level is relatively difficult, and most of the similar figure questions are tested and used. In recent years, it tends to combine quadratic function and quadratic equation; At the same time, reasoning is relatively difficult, and sometimes a lot of reasoning will be done to find and construct similar figures; In addition, the content of knowledge points is not low. Similar triangles are usually combined with circle, square, quadratic function, inverse proportion function and other knowledge points. To sum up, the background difficulty of similar figure is derailed from the other four dimensions. On the whole, the difficulty of the five dimensions is not balanced. The difficulties are mainly reflected in knowledge and skills. However, the background relative difficulty is on the rise in recent years. For example, the mirror reflection of the 16 questions in the 2021 Yiwu senior high school entrance examination adopts the scientific situation, which shows that the situation difficulty has been paid more and more attention in the senior high school entrance examination.\(^2\)
Generally speaking, as a common type of the final question, the similar figure questions are more difficult.

Figure 1 Comparison of relative difficulty of similar figures in senior high school entrance examination questions in Zhejiang Province from 2017 to 2021

Analysis on the difficulty of similar figure test

3. The following from the intuitive imagination, logical reasoning, mathematical operation dimensions, combined with the specific analysis of the embodiment of the difficulty of similar figure questions.

3.1 Difficulty analysis of visual imagination

Through the analysis of the eight test questions in Zhejiang Province, I found that the similar figure test questions tend to need to add auxiliary lines to construct the similar figure, so as to lay the foundation for the subsequent solution. In this way, looking for similar triangles requires not only intuitive imagination literacy, but also enough logical reasoning literacy. For example, in the question of Wenzhou high school entrance examination in 2020, the key is to find out the similarity between two triangles. According to the known condition, if students have strong intuitive imagination literacy, they are easy to think of adding auxiliary lines to get similar triangles. If the intuitive imagination literacy is not enough, it can be combined with logical reasoning, according to the topic needs to carry on the reverse deduction, think of adding auxiliary line. Another example is the 16th question of Huzhou senior high school entrance examination in 2020. It is more difficult to get two similar triangles by adding auxiliary lines, because it is difficult to associate the similarity of the other two unrelated triangles.

Therefore, when the topic needs to add auxiliary lines, the requirements for students’ intuitive imagination literacy will be greater, and the difficulty will be greatly improved.

3.2 Analysis of difficulties in logical reasoning and mathematical operation

In recent years, the reasoning steps of senior high school entrance examination questions in Zhejiang Province tend to increase, and the calculation is more difficult. For example, in the 26th question of Ningbo senior high school entrance examination in 2019, it is necessary to add auxiliary lines, set up unknowns, prove similarity, and then use the similar triangle property to bring the unknowns into the fractional equation to solve the unknowns. It is difficult to simplify the fractional equation, and it is also necessary to solve the quadratic equation of one variable after simplification. Step by step, it is time-consuming and laborious, so many students may choose to give up directly under the balance.[3]

The difficulty of logical reasoning lies in that it can’t be achieved overnight. Sometimes it takes several steps of reasoning to achieve “another village”. The difficulty of mathematical operation lies in simplification. Sometimes students can set up unknowns and set up equations, but they lack the courage and courage to simplify. They are scared by its complex form and give up solving problems.

4. Teaching suggestions based on similar figure difficulties

Geometry teaching should pay special attention to basic figures, especially similar triangles, with basic figures such as pagoda figure, 8-figured figure, 7-figured figure[2]. Students need to be proficient in recognizing basic figures, associate them with similar figures when they see them, and improve their intuitive imagination.

Therefore, teachers should pay attention to cultivating students’ sensitivity to the basic figures in daily teaching, and help them summarize the basic figures of similar triangles on the basis of students’ independent exploration. This is helpful for students to understand and remember the basic figure.

References: