Original Research Article

On Boeing 737-300 Wing Aerodynamics Calculations Based on VLM Theory

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ABSTRACT

In this paper, aerodynamics coefficients of Boeing 737-300 were calculated using VLM (vortex lattice method) theory. The wing was assumed to be planar and was divided into 6×6 panels, which were in the trapezoid shape. Aerodynamics lifting and moment coefficients were calculated. Also, center of pressure location was found using data from VLM and wing geometry. Comparisons between literature, finite wing theory and VLM theory were done. It was found that maximum lifting coefficient error between literature and VLM was about 4.0%. Moreover, that between finite wing theory and VLM was about 2.2%. Center of pressure location error between finite wing theory and VLM was about 0.5%.

Keywords: VLM; Finite Wing Theory; Lifting Coefficient; Moment Coefficient; Center of Pressure

1. Introduction

VLM (vortex lattice method) is a numerical method which aims to solve flow around wing bodies. The method is based on lifting line theory proposed by Prandtl[1] and is considered to be an extension of that method. However, compared with the lifting line theory which assumes one horseshoe vortex per wing, VLM theory utilizes a lattice of horseshoe vortices, as described by Falkner[2]. The number of vortices used varies with the required pressure distribution resolution, and with required accuracy in the computed aerodynamic coefficients. Detailed description of VLM theory in the context of aerodynamics is displayed by Katz & Plotkin[3], Bertin & Smith[4] and others[5,10-12].

The vortex lattice method is based on ideal flow theory (known as Potential flow) which is supposed to model real flow; however, this simplified model representation is good enough for most engineering applications. VLM theory neglects all viscous effects. Likewise, turbulence, dissipation and boundary layer effects are not considered at all. Nevertheless, induced drag can be estimated together with or without special cases of stall phenomena.

In this current study, VLM theory is applied to Boeing 737-300 wings with comparisons to finite-wing theory and relevant literature.

2. VLM Model

Boeing 737-300 aircraft wing geometry is illustrated in Figure 1.

While technical data of the airplane has been obtained using Boeing Commercial Airplanes[6]. In order to use VLM theory properly wing extension was done as shown in Figure 2. VLM model assumptions are as follows:

- The flow field is incompressible, inviscid and irrotational.
- Lifting surfaces are thin.
- The angle of attack and the angle of sideslip are both small.
- Planar wing.
- Symmetrical wing shape around x-z axis, so it can be assumed that:

$$\Gamma(y) = \Gamma(-y) \tag{1}$$

- Symmetrical wing shape around x-z axis, so it can be assumed that:
- Panel shape is trapezoid.
- Trail vortexes are released in the wing plan, velocity direction or in the intermediate state since trail angle influence can be neglected.

Moreover, due to symmetry assumption all calculations were made on the right-wing side. In other words, total numbers of equations are for N/2 collocation points. However, B.C. was calculated with both wing side vortexes. In more detailed describe, the wing was divided into two main trapezoids such as each trapeze was divided into six parts in x direction and three parts in y direction, as shown in Figure 2. Now, VLM algorithm will be elaborated as follows:

- The Wing was divided into 6×6 panels.
- Horseshoe vortexes were placed as shown in Fig. 2 inside each panel according the following order:
- Coupled vortexes were placed at 1/4 panel chord point in y-axis direction such as symmetry profile fulfills that $C_m|_{C/4} = 0$
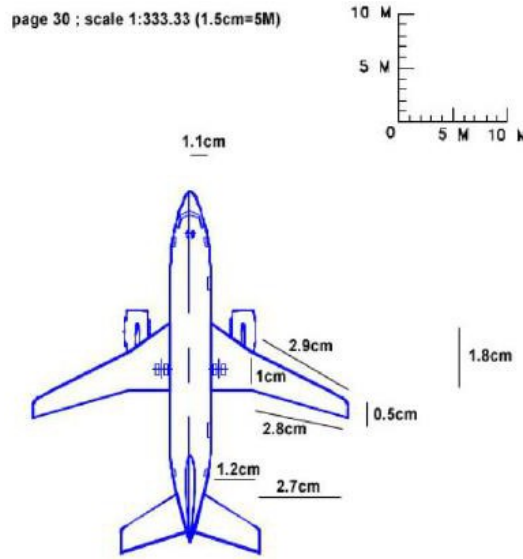- Traced vortexes were placed at the panels' boundaries.

Figure 1; Wing size.

- Half infinity vortexes were placed in x-axis direction.
- Collocation points were placed at 3/4 panel chord in the middle of the chord.
- Induced velocity on each collocation point in case of planar wing will be given according to Biot-Savart law[5] by the following equation which brought by Spurk and Aksel[7]:

$$w_n = \sum_{n=1}^{\infty} \frac{\Gamma_n}{4\pi} \left\{ \frac{1}{(x_m - x_{1n})(y_m - y_{2n}) - (x_m - x_{2n})(y_m - y_{1n})} \cdot \right.$$
$$\left[\frac{(x_{2n} - x_{1n})(x_m - x_{1n}) + (y_{2n} - y_{1n})(y_m - y_{1n})}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} \right.$$
$$\left. - \frac{(x_{2n} - x_{1n})(x_m - x_{2n}) + (y_{2n} - y_{1n})(y_m - y_{2n})}{\sqrt{(x_m - x_{2n})^2 + (y_m - y_{2n})^2}} \right] +$$
$$\frac{1}{y_{1n} - y_m} \left[1 + \frac{x_m - x_{1n}}{\sqrt{(x_m - x_{1n})^2 + (y_m - y_{1n})^2}} \right] \cdot$$
$$\left. - \frac{1}{y_{2n} - y_m} \left[1 + \frac{x_m - x_{2n}}{\sqrt{(x_m - x_{2m})^2 + (y_m - y_{2n})^2}} \right] \right\}$$

(2)

While $\left(x_{1n}, y_{1n}\right), \left(x_{2n}, y_{2n}\right), \left(x_m, y_m\right)$ represent A (initial point of the left half infinity vortex), B (initial point of the right half infinity vortex), C coordinates, respectively.

- In order to fulfill edge conditions, which means that normal velocity at the boundaries is zero, Neumann B.C. is applied such as:

$$\sum U_{norm} = -u_n \sin\left(\delta\right)\cos\left(\phi\right) - v_n \cos\left(\delta\right)\sin\left(\phi\right)$$
$$+ w_n \cos\left(\delta\right)\cos\left(\phi\right) + U_\infty \sin\left(\alpha - \delta\right)\cos\left(\phi\right) = 0 \qquad (3)$$

while α, δ, ϕ are attack angle, deflection angle and dihedral local angles, respectively. Also, u_n, w_n, v_n are the induced velocities in the normal directions, respectively. Where U_∞ is the free stream speed.

In our case of planar wing with no dihedral angle, where $\delta = \phi = 0$, set of N equations is obtained, by the form:

$$\sum U_{norm} = w_n + U_\infty \sin\left(\alpha\right) = 0 \qquad (4)$$

- Aerodynamics coefficients for planar wing are calculated as follows:
- Lifting coefficient per unit span function is of the form:

$$C_l(y) = \frac{2}{U_\infty c(y)} \sum_{N_x} \Gamma_j \qquad (5)$$

while c(y) is the chord function location.
- Total lifting coefficient function is of the form:

$$C_L = \frac{2}{S_w U_\infty} \sum_{N_x} \sum_{N_y} \Gamma_j \cdot \Delta y_j \qquad (6)$$

while Δy_j is local surface cell width. S_w is the wing area including the unexposed area.

- Pitch moment coefficient per unit span around wing vertex is calculated by:

$$C_{m,apex}(y) = -\frac{2}{c^2(y)U_\infty} \sum_{N_x} \Gamma_j \cdot x_{o,j} \qquad (7)$$

while $x_{o,j}$ is the distance between apex and vortex center.

- Total pitch moment coefficient around wing vertex is calculated by:

$$C_{M,apex}(y) = -\frac{2}{S_w U_\infty C_{ref}} \sum_{N_x} \Gamma_j \cdot x_{o,j} \cdot \Delta y_j \qquad (8)$$

Where the reference chord is the aerodynamic average chord and calculated by:

$$C_{ref} = \overline{c} = \frac{1}{S_w} \int_{-b/2}^{b/2} c^2(y)dy \qquad (9)$$

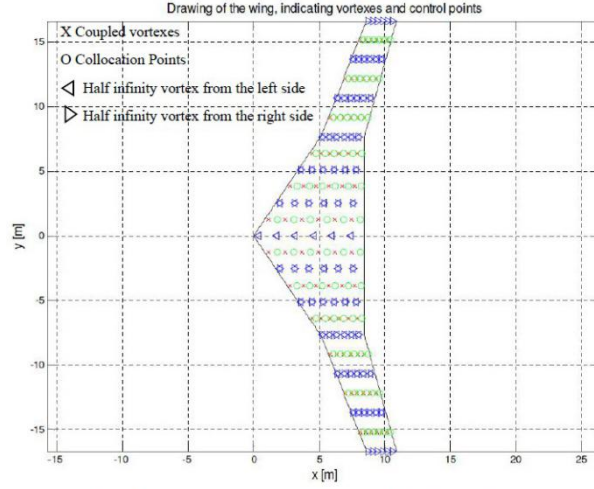While b is the total chord length.

Figure 2; Extension wing with vortex and collocation points partition.

- Center of pressure location is calculated using previous aerodynamic coefficients:

$$\bar{x}_{cp} = -\frac{C_{M_{\alpha,apex}}}{C_{L_{\alpha}}} - \frac{c_{y=0} - c_{ref}}{c_{ref}} \qquad (10)$$

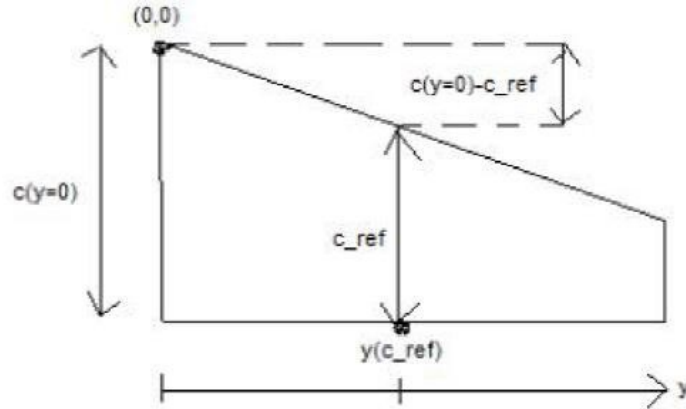While calculation was done according to Figure 3 around the theoretical point - wing apex (0,0).

Figure 3; Extension wing with vortex and collocation points partition.

In favor of completing this section, main geometrical parameters will be calculated using Boeing data sheet[6]. After measuring length chord, calculating wing trapezoid surface area and averaged aerodynamic chord together with rearward sweep angle, we got the following geometrical parameters:

$$b = 2 \cdot 16.7 = 33.4[m], S_w = 140.8[m^2],$$

$$AR = \frac{b^2}{S_w} = 7.9, C_{ref} \approx 5, \bar{\Lambda} = \tan^{-1}\left(\frac{8.3}{16.7}\right) \approx 27.7 \qquad (11)$$

while the total rearward sweep angle is dependent on y coordinate only. As a result, the angle calculation was based on simple triangle calculation. From here, Finite wing theory will be brought about by the current context.

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3. Finite wing theory

In the first step, finite-wing theory which are based on lifting line theory will be presented:

- Finite span.
- Vortices are being produced at the wing tips of the finite wing, which trail downstream.
- Downwash velocity.
- Mach number fulfills: $0 \leq M \leq 0.7$.
- Aspect ratio should fulfill $AR > 4$.

However, lift-curve slope in case of swept wing is given by Kuchemann[8]:

$$C_{L_\alpha}\Big|_{\text{Finite-Wing Theory}} = \frac{2\pi \cos \Lambda}{\sqrt{1 - M^2 \cos^2 \Lambda} + \dfrac{2 \cos \Lambda}{AR \cdot e}} \qquad (12)$$

While e is the taper ratio (also known as Oswald efficiency factor). Here, nominal Oswald efficiency (e is equal to 0.95 and Mach number (M) is equal to zero since we deal with in compressible flow. Additionally, taper ratio was calculated using Figure 4 as follows:

Figure 4. Swept wing geometry.
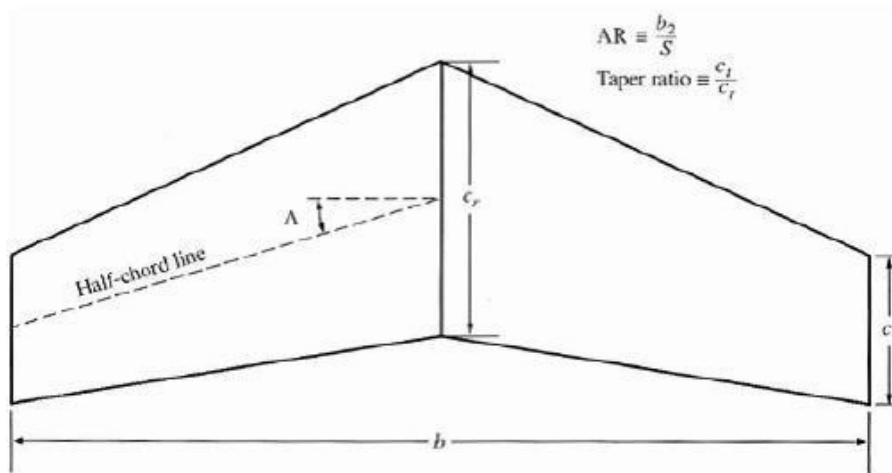
Where taper ratio is defined by:

$$e = \frac{c_t}{c_r} \qquad (13)$$

Next section, final results and comparison to finite-wing theory and literature will be presented and discussed.

4. Results & Discussion

Comparisons between VLM specific aerodynamics parameters, Finite-Wing theory and literature as appeared in Kuethe and Chow[9] are presented in Table 1.
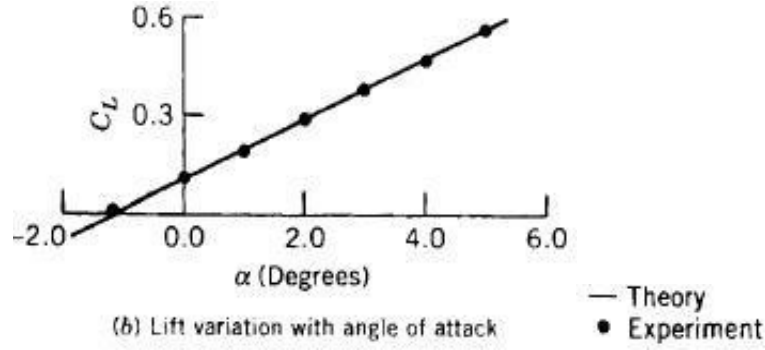
Figure 5. Lifting coefficient Vs. angle of attack using Boeing 737 wing data as given by Kuethe & Chow[9].

Slope lifting line calculation as brought by Kuethe & Chow[9] and shown in Figure 5, is calculated by:

$$C_{L_\alpha}\Big|_{\text{Literature}} = \frac{0.26 - 0.18}{\frac{\pi}{180}(2-1)} = 4.6\left[\frac{1}{rad}\right] \tag{14}$$

Parameters	VLM theory	Finite-Wing theory	Kuethe & Chow [12]	Error (ε)
$C_{L_\alpha}\left\lfloor\dfrac{1}{rad}\right\rfloor$	4.4	4.5	4.6	$\varepsilon_{\text{Literature Vs. Vlm}} = \dfrac{C_{L_\alpha}\big\|_{\text{Literature}} - C_{L_\alpha}\big\|_{VLM}}{C_{L_\alpha}\big\|_{\text{Literature}}}\cdot 100 = 3.9\%$ $\varepsilon_{\text{Finite Wing Theory Vs. Vlm}} = \dfrac{C_{L_\alpha}\big\|_{\text{Finite Wing Theory}} - C_{L_\alpha}\big\|_{VLM}}{C_{L_\alpha}\big\|_{\text{Finite Wing Theory}}}\cdot 100 = 2.2\%$
$C_{M_\alpha,apex}\left\lfloor\dfrac{1}{rad}\right\rfloor$	-4.7			
$\overline{x}_{cp}$	0.25 12	0.25		$\varepsilon_{\text{Finite Wing Theory Vs. Vlm}} = \dfrac{\overline{x}_{cp}\big\|_{VLM} - \overline{x}_{cp}\big\|_{\text{Finite Wing Theory}}}{\text{Finite Wing Theory}}\cdot 100 \approx 0.5\%$

Table 1. Aerodynamics parameter comparisons between VLM theory, Finite-Wing theory and literature (Kuethe and Chow, 1998).

Examination of table 4 leads to conclusion that VLM obtained results are coincided with finite-wing theory and literature. Likewise, lift slope error between VLM theory and finite wing theory is about 4%. Also, the error between VLM theory and literature is about 2%. Moreover, center of pressure location error between VLM theory and finite wing theory may also be neglected (about 0.5 %).

5. Conclusion

Boeing 737-300 finite wing aerodynamic coefficients were calculated using VLM theory compared to finite wing theory and literature. The wing was divided into N panels of the size: 6X6. The wing was assumed to be planar and the panels are in the trapezoid shape.

VLM theory is coincided with finite-wing theory and literature. Likewise, lift slope error between VLM theory and finite wing theory is about 4%. Also, the error between VLM theory and literature is about 2%. Moreover, center of pressure location error between VLM theory and finite wing theory may also be neglected (about 0.5 %).

Acknowledgements

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