ORIGINAL ARTICLE

Interest Rate Model with Humped Volatility under the Real-World Measure

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Abstract: The purpose of this paper is to develop real-world modeling for interest rate volatility with a humped term structure. We consider humped volatility that can be parametrically characterized such that the Hull–White model is a special case. First, we analytically show estimation of the market price of risk with humped volatility. Then, using U.S. treasury yield data, we examine volatility fitting and estimate the market price of risk using the Heath–Jarrow–Morton model, Hull–White model, and humped volatility model. Comparison of the numerical results shows that the real-world humped volatility model is adequately developed.

Keywords: Humped Volatility; Interest rate model; Interest-rate-risk management; Market price of risk; Real-world simulation

1. Introduction

Interest rate models have been used for option pricing under a risk-neutral measure, and financial institutions are increasingly interested in applying it for assessment of interest rate risk. For this purpose, the interest rate model should be used under the real-world measure. Such a model is called a real-world model.

To construct the real-world measure, it is necessary to estimate the market price of risk. In econometrics, the market price of risk has been estimated for short-rate models, such as in Dempster *et al*.[4] and Stanton[15]. In risk management, there are two main approaches to constructing the real-world measure: the so-called forward-looking and backward-looking approaches. Forward-looking approaches infer the real-world measure from market prices, such as option prices in Ross[14] (cf. Hull *et al*.[8]).

Backward-looking approaches estimate the market price of risk from the historical behavior of forward rates. This approach was used in Norman[13], where it was numerically estimated in the BGM model[3].Yasuoka[17,18,20] introduced a theoretical framework for real-world modeling in the LIBOR market model[9], the Heath–Jarrow–Morton (HJM) model[5], and the Hull–White model[7], respectively. Since the backward-looking approach is consistent with traditional methods of market risk management, practitioners find it simple to adopt. For example, van der Vlies[16] studies the evaluation of mortgage prepayment risk using the framework of [18].

The LIBOR market model is well-known as a standard model for derivatives pricing because of its practicality and positive interest rates. However, negative interest rates have been observed in some countries for a number of years. For such situations, the HJM model is practical as a scenario generator of interest rates. In particular, the Hull–White model, a special case of the HJM model, admits a mean reversion structure of volatility. The one-dimensional Hull–White model has been successfully applied to risk-management in practice and is of use in theoretical study.

doi: 10.18282/ff.v7i1.459

Because the Hull–White model admits a parametric volatility, it is convenient to estimate the market price of risk in this model, as shown in [20]. Taking advantage of this convenience, the correlation between the market price of risk and yields is studied in Yasuoka[19] for the Hull–White model under the real-world measure. Parametric volatility, as in the Hull–White model, is preferred in risk management because of its robustness. For example, in counterparty-credit-risk management, the potential future exposure of interest rate swaps can be evaluated by using the Hull–White model, as in [21].

However, the term structures of volatilities do not always satisfy mean reversion: the structure is sometimes a humped shape. See Kahn[10] and Heath *et al*.[6] for examples. Given the above circumstances, this paper aims to develop a real-world model with humped volatility, treating Hull–White volatility as a special case of humped volatility. There are various expressions of humped volatility, such as those in Agca[1], Mercurio and Moraleda[11], and Moraleda and Vorst[12]. Here, we adopt the mathematical representation used in [1].

Section 2 shows the estimation of the market price of risk in a humped volatility model. The argument used for this follows that in [20], where the real-world Hull–White model is constructed. Section 3 fits the volatility and estimates the market price of risk in the HJM model, Hull–White model, and humped volatility model, using U.S. treasury yield data. We examine the performance of volatility fitting and the estimation of the market price of risk by comparing results from these models.

2. Real-world model with humped volatility

This section briefly introduces real-world modeling with the Gaussian HJM model and develops this for the case of humped volatility. The argument basically follows that of [18,20].

2.1 The Gaussian HJM model

We denote by $f(t,T)$ the instantaneous forward rate (hereinafter, the forward rate) at time $t < T$ for maturity T. The instantaneous spot rate r is given by $r(t) = f(t,t)$. We denote by $\sigma(t,T)$ a d-dimensional volatility of $f(t,T)$, and we set $\upsilon(t,T) = -\int_t^T \sigma(t,u)du$. Let W_t be a d-dimensional Brownian motion under a real-world measure P and $\varphi = (\varphi_1, \ldots, \varphi_d)^T$ be the market price of risk, where the superscript T denotes transposition.

The dynamics of $f(t,T)$ is represented in the HJM model by

$$df(t,T) = \{-\sigma(t,T)\cdot\upsilon(t,T) + \sigma(t,T)\cdot\varphi(t)\}dt + \sigma(t,T)\cdot dW_t, \tag{2.1}$$

where $\cdot$ denotes the inner product in R^d. When the volatility $\sigma(t,T)$ is deterministic, the HJM model is called Gaussian. In this paper, we always work with the Gaussian HJM model, and further assume that $\sigma(t,T)$ is continuous in t and T. The state price deflator is a stochastic process satisfying

$$d\xi_t/\xi_t = -r(t)dt - \varphi(t)\cdot dW_t.$$

Using this, the no-arbitrage price is calculated under P as follows. Let C_T be the payoff at time T of some security. The no-arbitrage price of this option at time $t < T$ is given by

$$\frac{1}{\xi_t}E_t[\xi_T C_T],$$

where $E_t[\cdot]$ denotes the conditional expectation at t under P. For a small time step $\Delta t > 0$, $f(\Delta s, T)$ is represented by

$$f(\Delta s,T) = f(0,T) + \{-\sigma(0,T)\upsilon(0,T) + \sigma(0,T)\varphi(0)\}\Delta s + \sqrt{\Delta s}\sigma(0,T)W_1, \tag{2.2}$$

from the Euler integral, where $W_1 = \int_0^1 dW_s$. Naturally, we may identify W_1 with a d-dimensional standard normal distribution. This discrete expression is used for the estimation of the market price of risk and for the Monte-Carlo simulation of $f(\Delta s, T_i)$ with $i = 1, \ldots, n$.

Volatility and principal component analysis

We denote by $x = T - t$ the length of time from t until T. Let $x_1, \ldots, x_n$ be a sequence such that $x_i = \delta i$ for $0 \le i \le n$. Let $\Delta t > 0$ be fixed, and let $\{t_k\}_{k=1,\ldots,J+1}$ be a sequence of observation times, with $t_1 = 0$ and $t_{k+1} = t_k + \Delta t$, where $J + 1$ is the number of observations. In practice, we observe the forward rate $F(t_k, x_i)$ with a fixed time length x_i such that $F(t_k, x_i) = f(t_k, t_k + x_i)$. We define the change of F, keeping the maturity date $t_k + x_i$, such that

$$\Delta F_i(t_k) = F(t_{k+1}, x_i - \Delta t) - F(t_k, x_i).$$

Principal component analysis of the dataset $\{\Delta F_i(t_k)\}_{i,k}$ gives the lth eigenvalue ρ_l^2 and the lth principal component $e^l = (e_1^l, \dots, e_n^l)$ for $l = 1,2,\dots$. We assume without loss of generality that all eigenvectors are chosen such that $e_1^l > 0$ and $\rho_l > 0$. We assume that $\sigma(t,T)$ satisfies $\sigma^l(0,x_i) = \rho_l e_i^l$. For simplicity of notation, we abbreviate $\sigma(0,x_i)$ and $\upsilon(0,x_i)$ to σ_{0i} and υ_{0i}, respectively, for $i = 1, \dots n$.

Market price of risk in the Gaussian HJM model

Assuming that the market price of risk is constant during the sample period, the lth market price of risk φ_l is estimated by [18] such that

$$\varphi_l = \frac{1}{\rho}\sum_{i=1}^{n}\{E^H\left[\frac{\Delta F_i}{\Delta t}\right] + \sigma_{0i}\upsilon_{0i}\}e_i^l, \tag{2.3}$$

where $E^H[\,]$ denotes the sample mean. This solution is the maximum likelihood estimate, as shown in [20]. Monte-Carlo simulation of interest rates under P is referred to as real-world simulation. For numerically performing real-world simulation, we define the rolled trend of $F(\cdot, x_i)$ as $E^H\left[\frac{\Delta F_i}{\Delta t}\right]$, and the lth rolled trend score R_l as

$$R_l = \sum_{i=1}^{n} E^H\left[\frac{\Delta F_i}{\Delta t}\right] e_i^l. \tag{2.4}$$

The rolled trend represents the averaged change of the forward rate, reflecting the rolling effect, and R_l measures the change of the forward rate curve with respect to e^l. For details, see [18] or [20].

Equation (2.3) is represented as

$$\varphi_l = \frac{1}{\rho_l}\left\{R_l + \sum_{i=1}^{n} \sigma_{0i}\,\upsilon_{0i} e_i^l\right\}. \tag{2.5}$$

Property of real-world simulation

Once we obtain the value of φ, the real-world simulation can be performed from (2.2) by using

$$f(\Delta s, x_i) = f(0, x_i) + \{-\sigma_{0i}\upsilon_{0i} + \sigma_{0i}\varphi\}\Delta s + \sqrt{\Delta s}\sigma_{0i} W_1 \tag{2.6}$$

with time step Δs and each i. Since $|\sigma_{0i}\upsilon_{0i}|$ experimentally takes a small value, the drift term of the above is roughly dominated by the value of φ. For instance, when the market price of risk takes a large positive (resp., negative) value, then the real-world model predicts rising (resp., falling) interest rates.

Mean-price property

Let us consider a period A$= [0,T]$ and divide this into two subperiods $B_1 = [0, T/2]$ and $B_2 = [T/2, T]$. We denote by φ_A, φ_{B1}, and φ_{B2} the market price of risk in periods A, B_1, and B_2, respectively, and we denote the corresponding volatilities by σ_A, σ_{B1}, and σ_{B2}, again respectively. Assuming constant volatility through the whole period A, the following proposition holds. That this property, the "mean price property", holds is proven in [20].

Proposition 1. *We assume that $\sigma_{A1} = \sigma_{B1} = \sigma_{B2}$ in the matrix sense of equality. Then, it follows that*

$$\varphi_A = \frac{\varphi_{B1} + \varphi_{B2}}{2} \tag{2.7}$$

in the vector sense.

This relation roughly holds for actual data. Indeed, when we estimate the market price of risk in practice, the market price of risk φ_A of the whole period takes a roughly intermediate value between φ_{B1} and φ_{B2}. We shall see this feature in our numerical example.

2.2 Humped volatility model

For simplicity we assume that $\rho_l = 0$ for $l \geq 2$, that is, that the historical dynamics of forward rates are driven by only the first volatility component. Where it will not cause confusion, we omit the superscript l for the order in principal components in the following.

There are various expressions of humped volatility ([1,11,12], etc.). In this paper, we work with the expression of volatility proposed in [1]:

$$\sigma(t,T) = \sigma\{\gamma(T-t) + 1\}\exp\{-k(T-t)\}\ , \tag{2.8}$$

where σ, γ, and k are non-negative constants. It is known that this function is humped with respect to $T - t$ when $\gamma > k$. Using this parametric representation, the real-world model can be built more specifically. In particular, when

$\gamma = 0, \sigma(t,T)$ becomes

$$\sigma(t,T) = \sigma \exp\{-k(T-t)\} \ , \tag{2.9}$$

which is well known as the volatility of the Hull–White model in the HJM framework. Here, k is referred to as the mean reversion rate. The real-world modeling in the Hull–White model is introduced in [20], where argument is similar to the following. For convenience, we call the volatility of (2.9) the Hull–White volatility. We approximate the first volatility component ρe with σ, γ and k chosen such that

$$\rho e_i \quad \approx \sigma(\gamma x_i + 1)\exp(-k x_i). \tag{2.10}$$

For this purpose, consider the least-squares problem

$$\sum_{i=1}^{n}\{\rho e_i - \sigma(\gamma x_i + 1)\exp(-k x_i)\}^2 \tag{2.11}$$

under the restriction

$$\rho^2 = \sigma^2 \sum_{i=1}^{n}(\gamma x_i + 1)^2 \exp(-2k x_i).$$

(2.12)

The condition (2.12) is referred to as the norm-invariant condition; it ensures that the implied humped volatility and the first volatility component have the same norm. Solving this yields σ, γ and k. From these parameters, we set the volatility σ_{0i} such that

$$\sigma_{0i} = \sigma(\gamma x_i + 1)\exp(-k x_i); \ i = 1, \dots, n. \tag{2.13}$$

The approximation error in (2.10) is measured as the ratio of squared difference between the approximation to the first volatility component on average, such that

$$\frac{\{\frac{1}{n}\sum_{i=1,\dots,n}\{\rho e_i - \sigma(\gamma x_i + 1)\exp(-k x_i)\}^2\}^{1/2}}{\frac{1}{n}\sum_{i=1,\dots,n}\rho e_i}. \tag{2.14}$$

Since the function (2.11) is not globally downward convex, these parameters may not be unique. Volatility fitting is a practical matter, so non-uniqueness is not a severe problem in this paper. In practice, the above approximation error might help to solve the least-squares problem.

Next, we define an n -dimensional vector, $(\tilde{e}_1, \dots, \tilde{e}_n)$ by

$$\tilde{e}_i = \frac{\sigma}{\rho}(\gamma x_i + 1)\exp(-k x_i); \ i = 1, \dots, n. \tag{2.15}$$

From (2.12), we see that $\sum_{i=1}^{n}(\tilde{e}_i)^2 = 1$. Hence, we may regard $(\tilde{e}_1, \dots, \tilde{e}_n)^T$ as the first principal component, rather than $(e_1, \dots, e_n)^T$. Equation (2.13) is represented by

$$\sigma_{0i} = \rho \tilde{e}_i; \ i = 1, \dots n, \tag{2.16}$$

in which the humped volatility $(\sigma_{01}, \dots, \sigma_{0n})^T$ is defined anew as the first volatility component. We may estimate the market price of risk by (2.3) for the case of $\mathrm{d} = 1$.

In the Hull–White model, the parameters σ and k are determined in the same manner, letting $\gamma = 0$ in the above. For details, see [20]. The approximation error is similarly defined by (2.14), letting $\gamma = 0$.

Real-world model with humped volatility

From Agca[1], we have that

$$\begin{aligned} v(t,T) &= -\int_t^T \sigma\{1 + \gamma(u-t)\}\exp\{-k(u-t)\}du \\ &= -\frac{\sigma\gamma}{k}\left[\left(\frac{1}{\gamma} + \frac{1}{k}\right)[1 - \exp\{-k(T-t)\}] - (T-t)\exp\{-k(T-t)\}\right]. \end{aligned}$$

It follows that

$$v_{0i} = -\frac{\sigma\gamma}{k}\left[\left(\frac{1}{\gamma} + \frac{1}{k}\right)\{1 - \exp(-k x_i)\} - x_i \exp\{-k x_i\}\right] \ . \tag{2.17}$$

From (2.13) and (2.17), we can numerically calculate $\sum_{i=1}^{n}\sigma_{0i}\, v_{0i}\tilde{e}_i$. The market price of risk is estimated from (2.3).

We finally present a form for real-world simulation. Set $T_i = x_i$ for $i = 1, \dots, n$; here, T_i indicates the date and

x_i indicates the time length. Let $f(0, T_i)$ for $i = 1, \dots n$ be an initial forward rate. For a small time step Δs, we have

$$f(\Delta s, T_i) = f(0, T_i) + \sigma_{0i}(-v_{0i} + \varphi)\Delta s + \sqrt{\Delta s}\sigma_{0i}W_1. \tag{2.18}$$

3. Numerical examples

3.1 Data and volatility fitting

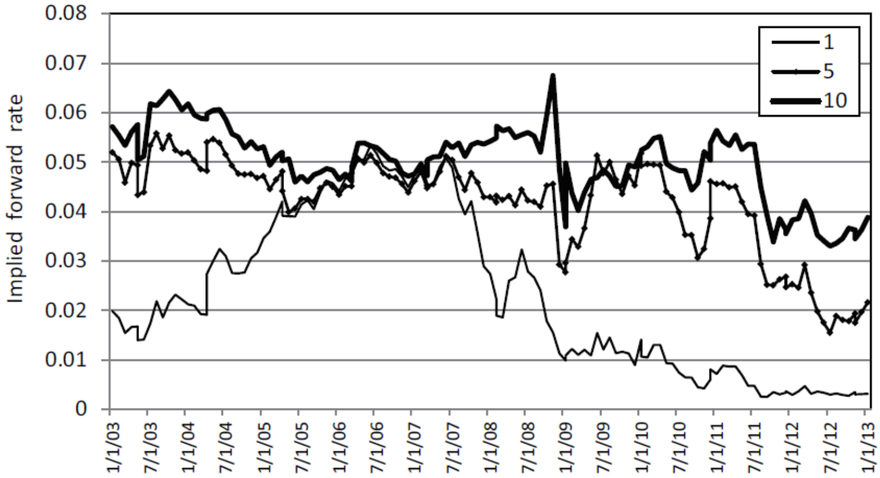

Figure 1. Forward rates in U.S. Treasury market, where the labels 1, 5, and 10 indicate the forward rate over the six-month periods beginning at 1, 5, and 10 years, respectively. Yield data were retrieved from [2].

We use U.S. Treasury yields from 10 January 2003 to 25 January 2013. Setting $\delta = 0.5$ (years) and $x_i = \delta_i$ for $\mathrm{i} = 1,2, \dots, 20(\mathrm{n} = 20)$, the 6-month forward rate is obtained for every four weeks in this period. **Figure 1** shows a historical chart of the implied forward rates. For our numerical examples, we split this sample period into two: period A is the first part, from 10 January 2003 to 4 January 2008, and period B is the last part, from 4 January 2008 to 25 January 2013. Period C is defined as the whole period, from 10 January 2003 to 25 January 2013.

Figure 2 shows the forward rate curves of the three dates that bound the periods. From this, we see a flattening of the forward rates in period A, bull-steepening in period B, and falling in period C. The first rolled trend score R takes a small negative value for period A, and a larger negative value for periods B and C. From (2.5), we may expect that the value of the first market price of risk is negative in period A and strongly negative in periods B and C. For details of this qualitative estimation, see [20].

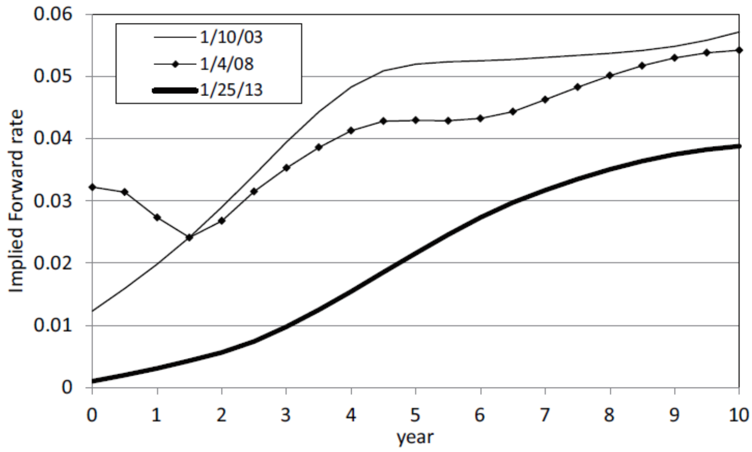

Figure 2. Implied forward LIBOR curves at three days (10 January 2003, 4 January 2008, and 25 January 2013).

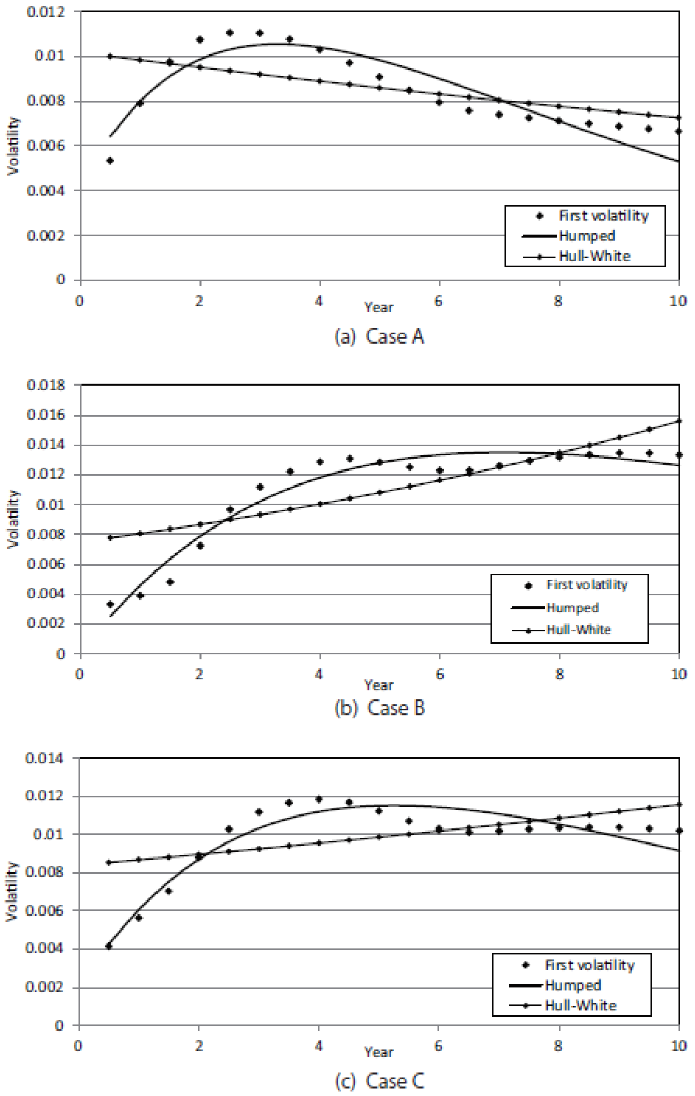

Figure 3. First volatility component with Hull–White volatility and humped volatility

3.2 Volatility fitting

For convenience, we regard the 6-month forward rate as the instantaneous forward rate in our numerical analysis. Setting $\Delta t = 28/365$ (i.e., four weeks), we obtain the volatility components in the HJM model by principal component analysis.

In the following, the numerical examples for periods A, B, and C are referred to as Cases A, B, and C, respectively. The first volatility components are approximated by the Hull–White volatility and by the humped volatility as explained in Section 2.2. Table 1 lists the volatility parameters and approximation error for Cases A, B, and C. In the table, "contribution rate" indicates the contribution rate of the first principal component. We see that the first volatility component explains more than 70% of the covariance for all cases. **Figure 3** compares the volatilities of the three cases.

The first volatility component is marked with a dotted curve in **Figure 3**. From this, the term structure of volatility admits a humped shape in Case A. In Case B, the volatility component is upward convex, rising to the right. Since period C is the direct sum of periods A and B, the volatility structure shows an intermediate shape between that of Case A and that of Case B. It is difficult to approximate these term structures when using Hull–White volatility since the Hull–White volatility is downward convex.

In the humped volatility model, the approximation error is in the range 0.069–0.088 for all cases, which shows that the humped volatility works well for all cases. In the Hull–White model, the mean reversion rate k is negative in Cases B and C since the first volatility component roughly rises to the right. The approximation error is in the range 0.174–

0.195 for three cases, which is obviously worse than the error when using humped volatility. Therefore, humped volatility approximates the first volatility component better than Hull–White volatility does for our sample. These features are visually verified in **Figure 3** for all cases.

		Case		
		A	**B**	**C**
Start day		10/1/2003	4/1/2008	10/1/2003
End day		4/1/2008	25/1/2013	25/1/2013
Contribution rate		0.823	0.725	0.738
Humped volatility	**k**	0.249	0.141	0.177
	$\boldsymbol{\sigma}$	0.0043	0.0001	0.0021
	$\boldsymbol{\lambda}$	1.397	40.0	2.50
	Error	0.088	0.073	0.069
Hull–White volatility	$\boldsymbol{k}$	0.0337	-0.073	-0.0320
	$\boldsymbol{\sigma}$	0.0102	0.0075	0.0084
	Error	0.174	0.195	0.174

Table 1. Volatility parameters of three cases.

3.3 Estimation of the market price of risk

In the Gaussian HJM model, the market price of risk is estimated for each case by (2.5), using an eight-dimensional model. In the humped volatility model and the Hull–White model, the market price of risk is estimated by the method described in Section 2.2.

Table 2 compares the market price of risk among the three volatility structures for each case, where "HJM" means the first market price of risk as estimated by the eight-dimensional HJM model, and the values in parentheses represent the difference of the market price of risk from "HJM" in percentage. The market price of risk in "HJM" is -0.392 in Case A and -0.701 in Case B. Section 3.1 suggests that the market price of risk should be negative and larger in Case B than in Case A. The market price of risk is -0.594 in Case C, which is almost an intermediate value between Cases A and B. This is roughly explained by the mean price property, introduced in Section 2.1.

In both the humped volatility model and the Hull–White model, the market price of risk is close to the value of "HJM". The difference is less than about 1% in all cases. Consequently, there is no remarkable difference in the estimation of the market price of risk among volatility structures. We see that the market price of risk is adequately estimated in the humped volatility model.

Case	HJM	Humped volatility	Hull-White
A	-0.392	-0.395 (0.83)	-0.389 (-0.76)
B	-0.701	-0.697 (-0.57)	-0.710 (1.30)
C	-0.594	-0.591 (-0.45)	-0.591 (-0.35)

Table 2. Market price of risk in three cases.

"HJM" indicates the first market price of risk as estimated by the eight-dimensional HJM model, and numbers in parentheses are the percent difference from "HJM".

4. Conclusion

Here, we presented a practical method for real-world modeling of humped volatility. This model is a generalization of the real-world Hull–White model. Next, we showed numerical examples using U.S. treasury yields from 2003 to 2013. In this period, the term structure of the first volatility component was humped or upward convex. The humped volatility model showed better performance than the Hull–White model on volatility.

The first market price of risk was estimated by using the first volatility component in the Gaussian HJM model, the humped volatility, and the Hull–White volatility. Comparing these values among three volatility structures, there were only small differences. Consequently, we see that the market price of risk is reasonably estimated in the humped volatility model. And the real-world humped volatility model has been practically introduced.

Decralation of interest section

The author reports no conflicts of interest. The author alone is responsible for the content and writing of the paper.

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