On the Cultivation of Mathematical Modeling Thought in Junior High School Mathematics Teaching

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Abstract: The teaching of mathematical modeling runs through the essence of quality education from beginning to end. It can cultivate students' awareness and ability of applying mathematical concepts, principles and methods to solve problems in their lives, and offer students a new experience of learning mathematics, thus truly promoting the development of students' mathematical literacy.

Keywords: Junior High School Mathematics; Modeling Thought; Classroom Teaching

1. Introduction

Mathematical modeling is the bridge between mathematics and real life, and also an important method to cultivate students’ interest in learning and enlighten students’ minds. However, in the current junior high school mathematics classroom teaching, there are many problems, such as weak ability to solve practical problems by applying the learned knowledge, and lack of training strategies of mathematical “modeling literacy”. Therefore, it is of great significance to strengthen modeling thought in junior high school mathematics classroom teaching, help students experience the process of applying and learning mathematics. It is also important to effectively encourage them to explore and learn mathematical modeling, and continuously improve their mathematical modeling literacy.

2. The main ideas of junior high school mathematics modeling

Due to the differences in analysis methods, modeling purposes and practical problems, the types of models are also different. However, in practice, the following ideas can be adopted, as shown in Figure 1.

Figure 1. Thinking flow chart of junior high school mathematics modeling.

2.1 Analyze problems

Teachers should guide students to use mathematical language to express practical problems, and use the familiar objects and data around them to construct models. Specifically, in the process of analyzing practical problems, firstly, the mechanism analysis method is used to analyze the causal relationship of the problems and find out the laws reflect-
ing the internal mechanism. For example, when establishing the functional relationship between variables, the author analyzed the problems by means of proportional analysis. Secondly, test analysis method is adopted. Static and independent data should be analyzed by return analysis method, and dynamic data in variable relationship should be analyzed by time series analysis method. Thirdly, the simulation analysis method is adopted, that is, the analysis object is tested locally, and then modified repeatedly according to the results, so as to obtain the required structural model. For example, to verify the probability of coin appearance, the author applied computer simulation to analyze the problem.

2.2 Reasonable hypothesis

If a practical problem cannot be reasonably assumed, it is difficult to transform it into a mathematical problem. Therefore, teachers should guide students to make reasonable hypothesis about practical problems according to the purpose of modeling and the characteristics of actual objects. When making specific assumptions, firstly, it is necessary to analyze the data presented, and clarify the main purpose of modeling and the main characteristics of the object. Secondly, simplify the actual object, extract the main factors, and present the main problems of the actual object. Finally, the put forward hypothesis, and analyze, modify and supplement it in time, and then make decision according to the results of the model. It is worth noting that when reasonably supposing a problem, it not only requires to give full play to students’ observation and judgment, screen out the main factors of the problem, but also fully apply the knowledge of economy, physics and chemistry related to the problem. At the same time, the selected model function should be as simple as possible.

2.3 Common methods

In order to simplify complex problems and concretize abstract problems as much as possible, teachers should guide students to integrate and apply various methods, and transform the problems to be solved into familiar mathematical models. If similar problems have been solved, similar modeling methods should be applied. Moreover, it should be based on a certain equilibrium relationship existing in the process of material change, such as equation model. In addition, it is necessary to construct graphical models from the geometric point of view, and constantly use them to explain practical problems.

2.4 Model construction

In order to enable students to know what to do and why to do in the concrete model construction, teachers should guide students to constantly refine the problems according to the actual situation. Firstly, obtain data samples by stratified sampling and systematic sampling. Secondly, make full use of coordinate paper to draw scatter maps. If possible, modern information technology should be adopted to accurately draw scatter maps as much as possible. Thirdly, study the scatter maps drawn, analyze the variable relationship in it, and choose a similar model. Finally, verify and change the model, and make the constructed model conform to the actual situation to the maximum extent.

3. Cultivation of mathematical modeling thought in junior high school mathematics teaching

3.1 Skillfully modeling by life examples—creating a modeling atmosphere

Junior high school mathematics is more knowledgeable, but not interesting, especially for some abstract mathematics knowledge, which is difficult for students to understand only by oral explanation of teachers. Therefore, teachers should construct mathematical models with the help of life examples, cultivate students’ mathematical modeling thought, and enable them to solve complex mathematical problems with the help of simple and intuitive life examples, so as to better master mathematical knowledge. Modeling with life examples is more in line with students’ cognitive level, which is conducive to enhancing interest and making classroom teaching more effective.

For example, in the teaching of “addition and subtraction of rational numbers”, if the teacher only presents the formula “50+(-70)-10 = ( )”, students will only feel strange and boring. At this time, teachers can construct a mathe-
matical model by life examples, so as to enable students to understand the real life meaning of this formula and stimulate their interest. For example, a mathematical model can be constructed like this: Xiaoming’s school, home and bookstore are on the same street, with the school 20 meters south of Xiaoming’s home and the bookstore 100 meters north of Xiaoming’s home. One day, Xiaoming came out from home and walked 50 meters north, then -70 meters north, and finally, he walked 10 meters south. Now, where is Xiaoming? After modeling, students have a strong desire to explore the problem, and have devoted themselves to case analysis. Finally, the formula is listed as $50 + (-70) - 10 = -30$ m. From the formula, it can be known that Xiaoming is 30 meters south of his home. Mathematical modeling combined with life cases can quickly activate students’ thinking and improve the efficiency and quality of mathematics teaching.

3.2 Create multiple mathematical modeling situations—stimulate interest in modeling

In junior high school mathematics teaching, creating multiple mathematical modeling situations is a means to cultivate students’ modeling thought and methods, which can not only improve students’ mathematical thinking, but also help them improve problem-solving efficiency and improve the effectiveness of junior high school mathematics teaching. In specific teaching, teachers should cultivate students’ modeling thought by leading them to find out the differences of mathematical modeling types, such as equation modeling, inequality modeling, function modeling, and geometric modeling. Then, based on practical problems, by creating multiple modeling situations, students’ awareness of model building can be strengthened, so that they can flexibly adopt various modeling types to solve problems. Thus, teaching efficiency can be improved.

There is an example: there are 24 workers in the workshop, and 1400 screws and 2000 nuts should be produced at the same time, of which one screw should be matched with two nuts. If the number of screws and nuts produced every day just matches, how many workers should be assigned to produce screws and nuts? This topic seems to be complicated, and junior high school students often can’t get confused when they first come into contact with it. However, if teachers use the idea of equation modeling to create situations and cultivate students’ awareness of modeling, students can quickly solve the problems. The key of this topic lies in the application of the knowledge of one-dimensional linear equation. By analyzing the topic, students can set the number of screw production as $x$, and the number of nut production workers is $24 - x$. Combined with the equation “$2 \times 1400x = 2000 \times (24-x)$”, the final solution is $x = 10$. Then, the number of workers producing screws is 10, and that producing nuts is $22 - 10 = 12$. Through this idea, students’ problem-solving efficiency is greatly improved, and their modeling thought is cultivated.

3.3 Broaden the teaching method of modeling—encourage flexible modeling

In junior high school mathematics teaching, teachers need to combine the reality of students from different classes, extend and expand the original teaching methods, and adopt different means and methods, so as to better cultivate the mathematical modeling thought. Methods like Multimedia teaching, cooperative inquiry in primary students, problem heuristic teaching and flipping classroom can provide support for the cultivation of mathematical modeling thought. Teachers should select and use teaching methods flexibly based on different objectives and requirements, so as to encourage students to think and master mathematical modeling methods.

Case 1: In the study of “probability” related knowledge, teachers can organize students to explore in groups, and in this way encourage them to actively participate in learning activities. Since probability is widely used in life, teachers can create the following modeling for students: there are several identical balls in an opaque box, and each ball is marked with different numbers, namely 1, 2, 5, 6, 9 and 11. Xiaohua and Fengfeng take a small ball and records the numbers on the ball respectively. The question is: “What is the probability that the sum of the numbers of small balls taken by Xiaohua and Fengfeng is greater than 9?” On this topic, teachers can encourage students to cooperate in groups, use tables to explore and construct mathematical models. Because this method is very interesting, students can solve problems quickly.

In addition, teachers can organize students to cooperate and explore, strengthen the interaction between students, strengthen their modeling ability in this process, and improve their learning enthusiasm and participation. To explore
and prove that “the sum of the inner angles of a triangle is equal to 180°”, for example, as shown in Figure 2, the teacher first raised the question: “In primary school, everyone already knew that ‘the sum of the inner angles of a triangle is equal to 180°’. Do you still remember how to get this conclusion at that time? Can you find the correct reasoning method of the conclusion from the operation in Figure 1? How many reasoning methods do you have? Please draw a figure and explain it by reasoning.” Then the teacher should give an example: as shown in Figure 2, knowing △ABC, try to explain ∠A+∠B+∠C = 180°.

Since there is 180° in the conclusion to be proved in this question, and the knowledge about 180° in the students’ knowledge system only lies in the inner angle of triangle and the degree of straight angle, teachers can use the knowledge that students already have to construct a mathematical model. At this point, the teacher first leads the students to recall their previous knowledge, and uses jigsaw puzzles to find proof methods, then draws auxiliary lines, so that students can understand the role of auxiliary lines, and then naturally penetrates the model idea of “180°” in this process.

Case 2: As shown in Figure 4, it is known that circle O is the circumcircle of △ABC, and point o is the center of the circle. In △ABC, line AD is perpendicular to line BC, line BE is perpendicular to line AC, line AD and BE intersect at point H, line OG is perpendicular to line BC and intersect BC at point G. Try to prove: AH = 2OG.

Analyze the problem and construct a model: This topic is comprehensive, which combines two special graphics, namely triangle and parallelogram. It comprehensively covers the knowledge points such as the nature of circle, the median line of triangle, the nature of triangle, the judgment of parallelogram and the nature of parallelogram. The key to solve this question is how to construct a new mathematical model with auxiliary lines. Starting with two modeling
methods, knowledge of parallelogram and triangle knowledge is used to solve problems, which fully reflects the importance of establishing mathematical models in geometry application.

To prove:
As shown in Figure 5, draw the diameter CE of circle O crossing point O which intersect with circle O at points C and E, and connect EB and AE. It can be known that \( \angle CAE = 90 \).

That is, AE is perpendicular to AC and BH is perpendicular to AC.
\[
\therefore \text{EA} \parallel \text{BH}, \text{EB} \parallel \text{AH can be obtained by the same way.}
\]
\[
\therefore \text{quadrilateral AEBH is a parallelogram.}
\]
\[
\therefore \text{AH=EB}
\]

In \( \text{Rt}\triangle CEB \), OG \parallel EB, OC=OE
\[
\therefore \text{OG is the median line of } \triangle BCE, \text{OG=EB and AH=EB}
\]
\[
\therefore \text{OG=AH, that is, AH=2OG}
\]

Case 3: It is known that the three sides of \( \triangle ABC \) are a, b and c, and \( a^2+b^2+c^2 = ab+bc+ca \). Try to judge the shape of \( \triangle ABC \).

Solution: \( a^2+b^2+c^2 = ab+bc+ca, \)
\[
\therefore 2a^2+2b^2+2c^2 = 2ab+2bc+2ca,
\]
Therefore \( (a-b)^2+(a-c)^2+(b-c)^2 = 0 \)
\[
\therefore a = b = c
\]

That is, the shape of \( \triangle ABC \) is an equilateral triangle.

Analysis: Starting from the known formula, it will be very difficult to calculate the relationship between the corners of this triangle. Therefore, it is necessary to transform the geometric problem into an algebraic problem and work out a completely square formula.

Geometry knowledge in middle school is characterized by three-dimensional novelty, abstraction and variety, which is one of the difficulties for students to learn in middle school. However, once students have a clear and thorough understanding of the nature of the necessary mathematical models (such as triangles, parallelograms and circles), the abstract problems can be simplified and solved by constructing these simple mathematical models.

In the teaching process of this course, teachers can successfully design the process of “problem situation—model building” according to the teaching content, and simplify the problems in specific teaching, thus contributing to the effective development of students’ model thought and cultivation of the modeling awareness.

4. Conclusion

Mathematical modeling is a scientific method to solve problems. Introducing this idea into junior high school mathematics teaching can effectively improve students’ problem-solving efficiency and learning quality, increase the interest of mathematics teaching, arouse students’ enthusiasm, and encourage them to learn actively. Through this, the
development of students’ comprehensive ability in mathematics and their ability to solve practical problems with mathematical knowledge can be promoted.

References